Storage and perpendicular retrieving of light pulses in electromagnetically induced transparency media

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Abstract. Storing and perpendicular retrieving of light pulses in electromagnetically induced transparency media is analyzed. A simple obvious analytical solution for the retrieved new field is obtained. It is shown that the generated new pulse time-shape is governed by the initial probe pulse profile while the profile of the new pulse is governed by the time-shape of the initial probe pulse.

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Electromagnetically induced transparency (EIT) is a coherent interaction process where a coupling laser field is used to modify the optical properties of an atomic medium for probe laser field [1–3]. Since its discovery, great attention has been paid to EIT, caused by its many unique applications and effects, such as ultraslow light propagation [4,5], light storage [6–9], coherent control of temporal pulse shaping [10], etc. By using EIT also, the traditional effects of nonlinear optics may be enhanced [11,12]. The two dimensional light pulse storage for atomic beams was discussed in [13]. The possibility of changing the direction of the pulse by control beam direction altering has been recently numerically analyzed [14].

The theoretical study presented in this paper is limited to a particular case of perpendicularly propagating control and retrieving pulses. In this regime a simple and visual analytical solution is obtained and analyzed in detail. It is shown that in addition to fundamental interest, it can find many practical application. Particularly in determination of temporal shape of short pulses, or a pulse intensity profile, and in revealing the spatial properties of light storage.

The common EIT system is a medium of three level atoms, with two metastable lower states $(A \text{ system})$, interacting with two resonant laser fields tuned to two photon resonance (see Fig. 1). State $|2\rangle$ resonantly connects to state $|3\rangle$ by the coupling field E_{c1} = $A_{c1} \cos (k_{c1}x - \omega_{c1}t + \varphi_{c1}),$ and state $|1\rangle$ resonantly connects to state $|3\rangle$ by the initial probe field E_p = $A_p \cos(k_p x - \omega_p t + \varphi_p)$ tuned to the exact two-photon resonance. After turning off interaction certain coherence between the metastable states is distributed in the me-

Fig. 1. Level scheme of atoms with fields propagation directions. State $|2\rangle$ connects to state $|3\rangle$ by the coupling fields Ω_{c1} and Ω_{c2} . State $|1\rangle$ connects to state $|3\rangle$ by the probe field Ω_p and the generated new field Ω_n . Ω_{c1} and Ω_{c2} propagate in perpendicular directions. The field Ω_{c2} is turned on when Ω_{c1} has been turned off.

dia. Due to the coherence kept by the media turning on the coupling field $E_{c2} = A_{c2} \cos(k_{c2}y - \omega_{c2}t + \varphi_{c2})$ generates the new field $E_n = A_n \cos(k_n y - \omega_p t + \varphi_n)$. In the presented work a case of perpendicular E_{c2} and E_{c1} propagation directions has been analyzed. The field E_{c2} is assumed to turn on at $t = \tau_1$ when E_{c1} has been turned off already. For simplicity the fields E_{c1} and E_{c2} are assumed to be polarized along z-axis and wave vectors of probe and the storing field are assumed to be equal $k_p = k_{c1}$. So the system Hamiltonian in rotating wave approximation for $t < \tau_1$, when only probe E_p and the storing field E_{c1} are turned on is, as follows:

$$
H = \hbar \Delta \sigma_{33} - \hbar \Omega_p \sigma_{31} - \hbar \Omega_{c1} \sigma_{32} + H.c \tag{1}
$$

where $\Omega_i = A_i \mu_{3i} / \hbar$ are Rabi frequencies, $\sigma_{ij} = |i\rangle\langle j|$ are the atomic transition operators, $\Delta = \omega_{31} - \omega_p$.

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With the Hamiltonian (1) the atom dynamics is described by the Bloch equation:

$$
\dot{\rho} = -\frac{i}{\hbar} \left[\rho H \right] + A\rho,\tag{2}
$$

where Λ is a dissipation matrix, which describes decoherence and spontaneous emission. It is assumed,

$$
|\Omega_p| \ll |\Omega_{c1}| \text{ and } |\Omega_{c1}| \ll 1 \tag{3}
$$

which means that the probe pulse is weak compared to storing pulse, and the interaction is supposed to be adiabatic, T is the probe pulse duration.

The propagation equation for the Rabi frequencies Ω_p and Ω_{c1} may be written as

$$
\left(\frac{\partial}{\partial x} + \frac{1}{c}\frac{\partial}{\partial t}\right)\Omega_p = iq_p\rho_{31},
$$
\n
$$
\left(\frac{\partial}{\partial x} + \frac{1}{c}\frac{\partial}{\partial t}\right)\Omega_{c1} = iq_c\rho_{32},
$$
\n(4)

where $q_i = 2\pi \omega_i \mu_{3i}^2 N/\hbar c$, N is the atomic number density.

To the lowest nonvanishing order with respect to (3) the master equation (2) reduces to

$$
\rho_{21} = -\frac{\Omega_p}{\Omega_{c1}}, \quad \rho_{31} = -\frac{i}{\Omega_{c1}^*} \frac{\partial}{\partial t} \rho_{21}, \n\rho_{11} = 1, \quad \rho_{22} = \rho_{33} = \rho_{32} = 0,
$$
\n(5)

where assumption is made that all atoms of the medium are initially in the state 1.

By substituting (5) into the Maxwell equation (4) one gets the pulses propagation equations:

$$
\left(\frac{\partial}{\partial x} + \frac{1}{c}\frac{\partial}{\partial t}\right)\Omega_p = -\frac{q_p}{\Omega_{c1}^*}\frac{\partial}{\partial t}\frac{\Omega_p}{\Omega_{c1}}
$$

$$
\left(\frac{\partial}{\partial x} + \frac{1}{c}\frac{\partial}{\partial t}\right)\Omega_{c1} = 0.
$$
(6)

Solution for (6) can be easily found $[6]$:

$$
\Omega_p(t, x, y) = \Omega_{c1} (t - x/c)
$$
\n
$$
\times f\left(x - \int_0^{t - x/c} \frac{|\Omega_{c1}(t')|^2}{q_p} dt'\right) a(y), \quad (7)
$$
\n
$$
\Omega_{c1}(t, x) = \Omega_{c1}(t - x/c).
$$
\n(8)

In (7), $f(x)$ is determined by time shape, $a(y)$ by intensity distribution of the initial probe pulse. The coupling pulse width is assumed to exceed the probe pulse width so that the coupling pulse profile intensity distribution can be neglected.

The coherence propagation dynamics can be found by substituting equations (7) and (8) into (5) :

$$
\rho_{21} = -f \left(x - \int_{0}^{t - x/c} \frac{\left| \Omega_{c1} \left(t' \right) \right|^{2}}{q_{p}} dt' \right) a \left(y \right). \tag{9}
$$

Equations (7), (8) describe the well-known propagation of probe pulse with nonlinear group velocity $|\Omega_{c1}|^2/q_p$. In particular, if the coupling pulse is turned off, the probe's group velocity tends to zero, but if the coupling pulse is turned on again (retrieving pulse), probe pulse can be obtained since the coherence ρ_{21} stores information about the initial probe.

Let's now consider the case when the retrieving coupling is turned on perpendicularly to the initial pulse propagation direction. It is supposed that the retrieving coupling pulse Ω_{c2} turns on after turning off the storing pulse Ω_{c1} ($\Omega_{c2} \neq 0$ for $t > \tau_1$). The system Hamiltonian for $t>\tau_1$ is

$$
H = \hbar \sigma_{33} \Delta - \hbar \Omega_n \sigma_{31} - \hbar \Omega_{c2} \sigma_{32} + H.c.
$$
 (10)

It is obvious that the new Hamiltonian (10) of the system is similar to the Hamiltonian (1), so the system dynamics is governed by the same solution, which has different initial conditions. Note that there is no new field Ω_n at the medium entrance but because of the light storage process the medium has a certain coherence ρ_{21} . The propagation direction of the generated new field is the same as that of Ω_{c2} , according to the momentum conservation law and the condition $k_p = k_c$ [14]. The new field and coherence propagation equations can be obtained by the same procedure as above.

$$
\Omega_n = \Omega_{c2} \left(t - y/c \right) g \left(y - \int_{\tau_1}^{t - y/c} \frac{\left| \Omega_{c2} \left(t' \right) \right|^2}{q_p} dt' \right) b(x) \tag{11}
$$

$$
\rho_{21} = -g \left(y - \int_{\tau}^{t-y/c} \frac{|\Omega_{c2}(t')|^2}{q_p} dt' \right) b(x) \tag{12}
$$

where g describes the temporal shape of the generated new field and $b(x)$ describes the intensity profile distribution of the generated field, it has again been assumed that coupling pulse is wider than the probe, and the coupling pulse profile distribution is neglected. Note that

$$
\int_{\tau_1}^{t-y/c} \frac{\left|\Omega_{c2}\left(t'\right)\right|^2}{q_p} dt' = 0 \text{ for } t < \tau_1 \tag{13}
$$

since Ω_{c2} turns on later than τ_1 , and

$$
\int_{0}^{t-x/c} \frac{\left|\Omega_{c1}\left(t'\right)\right|^{2}}{q_{p}} dt' = \int_{0}^{\tau_{1}-x/c} \frac{\left|\Omega_{c1}\left(t'\right)\right|^{2}}{q_{p}} dt' \text{ for } t \ge \tau_{1} \tag{14}
$$

since Ω_{c1} turns off by the moment τ_1 .

Now, by comparing (12) and (9) at the moment $t =$ τ_1 and taking into account (13) and (14), one obtains the time shape and intensity profile distribution of the

Fig. 2. Time shapes of storing and retrieving pulses (a) $\Omega_{c1}T = \{t > 8, 10e^{-(t/T-8)^2}, t < 8, 10\}$, (solid) $\Omega_{c2}T = \{t < 15, 10\}$ (dotted) The initial grade pulse grafie (a) $e^{-(y/20x_0-5)^2}$ (b) and terms and change $15, 10e^{-(t/T-8)^2}, t> 15, 10$ (dashed). The initial probe pulse profile $a(y) = e^{-(y/20x_0-5)^2}$ (b), and temporal shape $f(t)\Omega_c(t)T =$ $0.9e^{-(t/T-2.5)^2} + 1.2e^{-(t/T-6)^2}$ (c).

generated new field:

$$
g(y) = a(y)
$$

$$
b(x) = f\left(x - \int_{0}^{\tau_1 - x/c} \frac{\left|\Omega_{c1}(t')\right|^2}{q_p} dt'\right).
$$

This means that the obtained new field has an interchanged time shape and intensity profile distribution with the initial probe field. Or, in other words, temporal shape of the generated new field is governed by intensity profile distribution of the initial probe and intensity profile distribution of the generated new pulse is governed by the initial pulse temporal shape. Finally, a simple obvious solution can be obtained for the new pulse propagation.

$$
\Omega_n = \Omega_{c2} \left(t - y/c \right) a \left(y - \int_{\tau_1}^{t - y/c} \frac{|\Omega_{c2} \left(t' \right)|^2}{q_p} dt' \right)
$$

$$
\times f \left(x - \int_{0}^{\tau_1 - x/c} \frac{|\Omega_{c1} \left(t' \right)|^2}{q_p} dt \right) \tag{15}
$$

where

$$
\Omega_{c2}\left(t-y/c\right)a\left(y-\int\limits_{\tau_{1}}^{t-y/c}\frac{\left|\Omega_{c2}\left(t'\right)\right|^{2}}{q_{p}}dt'\right)
$$

describes the temporal shape of the new field where $a(y)$ is initial profile distribution of the probe and

$$
f\left(x-\int\limits_{0}^{\tau_1-x/c}\frac{\left|\Omega_{c1}\left(t'\right)\right|^{2}}{q_p}dt\right)
$$

describes profile intensity distribution of the new field where $f(t)$ is the time-shape of the initial probe. Equation (15) shows that measuring the time-shape of the generated field gives us information about the initial probe

pulse profile and vice versa measuring the generated pulse profile gives us information about the time-shape of the initial probe pulse.

Below is introduced the probe field, the new field and coherence spatial distributions at different time moments. The time shapes of storing pulse and retrieving pulse are assumed as $\Omega_{c1}T = \{t > 8, 10e^{-(t/T - 8)^2},\}$ $t \le 8, 10$, $\Omega_{c2}T = \{t \le 15, 10e^{-(t/T-8)^2}, t > 15, 10\}$ (Fig. 2a). The probe pulse profile and the temporal shape are taken as $a(y) = e^{-(y/20x_0-5)^2}$, $f(t)\Omega_c(t)T =$ $0.9e^{-(t/T-2.5)^2} + 1.2e^{-(\frac{t}{T}-6)^2}$ respectively (Figs. 2b, 2c), where $x_0 = (q_p T)^{-1}$. Spatial distributions of coherence and probe for different moments of time are depicted in Figure 3. As can be seen, probe pulse propagates along the x-axis and is absorbed when coupling pulse Ω_{c1} is turned off (Figs. 3a–3c). But due to the coherence pumped to the media (Fig. 3c), the new pulse can be generated by switching on the retrieving pulse Ω_{c2} (Figs. 3d–3f). Since coherence is distributed along both the x - and y -axis, even when retrieving pulse turns on along the y-axis, new pulse is obtained. As stated above, the propagation direction of the generated new pulse coincides with the propagation direction of the retrieving pulse and the time-shape is governed by the initial probe pulse profile, while the profile of the new pulse is governed by the time-shape of initial probe pulse.

For a 1 cm long ⁸⁷Rb cell at the atomic density of 10^{14} atoms/cm³ if probe and coupling are applied at the Zeeman sublevels of $5S_{1/2}$, $F = 2$, $\longrightarrow 5P_{1/2}$, $F = 1$ transition $x_0 \approx 0.002$ cm for $T = 10^{-8}$ s. The spatial length L of the stored pulse is determined by the condition $q_pLT \sim (\Omega_{c1}T)^2 \approx 100$ (according to Eq. (3)), i.e., $L \sim 0.1$ cm. Then the experiment can be conducted with 0.1 cm laser beams. In the case of solid state media, due to the increase of atomic number density $N \sim 10^{18}$ [9], the experiment can be conducted with pulse duration of the psec order with the same laser beams.

In conclusion, it is shown that propagation control of the pulses in optically thick media can be used for producing interchange between the pulse time-shape and intensity profile distribution. This mechanism is based on

Fig. 3. Probe field and coherence spatial distribution at the different moments of time. Probe pulse is absorbed when storing pulse Ω*c*¹ is turned off (a– c), but due to the metastable levels coherence pumped into the media, the new pulse is generated after turning on the retrieving pulse Ω_{c2} (d–f). The propagation direction of the generated new pulse coincides with the propagation direction of the retrieving pulse (along y). The time-shape of generated pulse is governed by the initial pulse profile, and the profile of new pulse is governed by the time-shape of the initial pulse.

light storage and retrieving process in the case of perpendicular propagation directions of storing and retrieving pulses. This effect can find application both in measuring short pulse time-shape and profile. Analytical solution for the generated new field is obtained and numerical results describing the dynamics of the system in detail are presented.

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